DEPARTMENT OF EDUCATION

Benchmark Achievement Level Descriptors (Benchmark ALDs)

Overview and Purpose

The development of Achievement Level Descriptors (ALDs) is a critical step in communicating student performance in terms of levels or categories of performance on any standardized assessment. For Minnesota Comprehensive Assessments (MCAs), ALDs are developed in collaboration with educators during the first year of full implementation. The ALDs provide a description of grade-level student performance on MCAs for each of the achievement levels of Exceeds the Standards, Meets the Standards, Partially Meets the Standards, and Does Not Meet the Standards. These statements are included on a student's MCA score report to aid families in score interpretation. More detail regarding the development of the MCA Achievement Level Descriptors is on Testing 1, 2, 3.

Over the years, educators have requested more specific descriptions of the knowledge, skills and abilities of students who typically score in each of the different MCA achievement levels beyond what the traditional ALDs offer. In response to this need, Minnesota Department of Education (MDE) staff collaborated to outline more specific descriptions, the Benchmark ALDs for Mathematics and Reading. The purpose of Benchmark ALDs is to

1. promote equity for all students across the state by clarifying expected learning outcomes for instruction and local assessment of Minnesota Academic Standards in Reading and Mathematics; and
2. support teachers' analysis of the depth of their curriculum, instruction, and classroom assessments.

The Mathematics and Reading Benchmark ALDs were developed by

- reviewing test questions and test data for all operational MCA III questions, in many cases 800–1,000 questions per grade;
- grouping items within each benchmark based on student performance on the items relative to their overall performance on the MCAs; and
- reviewing the achievement level groupings of questions within each benchmark for commonalities in the skills, understanding and context needed to correctly answer the items. Each Benchmark ALD describes some of the skills typically demonstrated by students whose overall performance on the MCAs is at that achievement level. These skills are in addition to the descriptions at the lower achievement levels.

Released Examples

Where possible, released examples that illustrate skills described in the benchmark and achievement level are listed in the document. To view examples, click on "Released Example" in the Benchmark ALD tables or go to the Minnesota Question Tool (https://public.education.mn.gov/nqt/). Once at the Minnesota Question Tool (MQT) site, you can enter or copy and paste the released example identification number into the "Search by Question ID" field. Note that within the MQT you can find additional questions that are aligned to the academic standards but are not specifically listed in the Benchmark ALD tables.

Example items are not currently available for all benchmarks and achievement levels in the Benchmark ALD tables. MDE will update the document as more released examples become available.

Training Module

Watch the training module to learn how to use the Benchmark ALDs to evaluate the rigor of classroom assessments and instructional materials: https://testing123.education.mn.gov/test/plan/success/.

This module will help educators understand how the Benchmark ALDs can be used to facilitate the learning outcomes defined in the Minnesota K–12 Academic Standards in Mathematics and Reading and to evaluate the rigor of classroom assessment and instruction.

Grade 11 Mathematics Benchmark Achievement Level Descriptors

Algebra

Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate. (9.2.1)

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9.2.1.1 **Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain.**	Uses function notation to evaluate linear and simple quadratic functions to find an output value, *f(a)*, by substituting a given number "*a*" in for *x* when *x* is an integer greater than zero	Evaluates quadratic, exponential, and simple rational equations using function notation and for a given positive or negative integer value(s) and positive decimals to the tenths place in the domain	Evaluates various functions with positive and negative decimal and fraction inputs as well as simple expressions	Evaluates complex functions using irrational values as well as composite functions

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9.2.1.2 **Distinguish between functions and other relations defined symbolically, graphically or in tabular form.**	Identifies whether the graph of a familiar equation type is a function	Identifies a point that can be added to a group of points to maintain the function Identifies a function when presented with up to 4 points in tabular form	Distinguishes between a function and a relation given as the graph of an equation, step function, and plotted points, or when given a list of ordered pairs Understands that a set of ordered pairs in which the *x* value repeats is not a function	Distinguishes functions and relations given in equation, graph, or table format Compares and justifies relations and functions **Released Example:** 501321
9.2.1.3 **Find the domain of a function defined symbolically, graphically or in a real-world context.**	Knows that the domain of a function is a range of possible values that may extend to infinity	Understands that the domain of a rational function excludes values which result in a value of zero in the denominator	Uses inequality symbols to define the domain of various functions, including rational, simple piecewise, and functions with vertical asymptotes	Identifies the domain of various functions including piecewise functions Identifies the domain restrictions of a function in context

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9.2.1.4 **Obtain information and draw conclusions from graphs of functions and other relations.**	Interprets a horizontal part of a graph in context Identifies linear graphs to show constant growth rates	Solves one- and two-step problems by reading and analyzing graphs of continuous functions and scatterplots presented in the first quadrant and in context Identifies an interval of values from a graph to satisfy a situation in context	Identifies correct interpretations of information presented in the graph of a piecewise, step, or quadratic function **Released Example:** 504031	Identifies the graph which models a situation in context, including step functions, involving quantities that change over time Defines the range of a function shown graphically

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9.2.1.5 **Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - h)^2 + k$, or in factored form.**	Identifies the location of the vertex when given the graph of a parabola Knows that the vertex and the axis of symmetry are related	Identifies the axis of symmetry when given a graph of a parabola Identifies the *y*-coordinate of the vertex when given a quadratic function in vertex form	Identifies the *y*-intercept, vertex, or axis of symmetry (for $x \neq 0$) of a quadratic function given graphically or as an equation in standard form or factored form Solves problems using a partial equation or graph of a parabola and the vertex **Released Example:** 502060	Calculates the *y*-intercept, vertex, or axis of symmetry (in the form $x = n$) of a quadratic function given graphically or as an equation in standard form or factored form Solves problems using only a partial graph of a parabola
9.2.1.6 **Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.**	Identifies the maximum and minimum values on the graph of a function Identifies the *y*-intercept on the graph of a function	Understands that the zeros of a function are *x*-intercepts and identifies multiple zeros of a function on a graph **Released Example:** 45019	Uses $a < x < b$ notation consistently to identify intervals over which a function is increasing or decreasing	Identifies the *y*-intercept, multiple zeros, multiple maxima and minima, and increasing and decreasing intervals of functions

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9.2.1.7 **Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.**	Knows that graphs of some functions approach but never reach some values	Identifies the vertical and horizontal asymptotes of a given parent function in graph format Knows that rational functions have an asymptote that is related to the denominator	Identifies the vertical, horizontal, or pair of asymptotes for a given rational function from an equation or a graph Knows that exponential functions have an asymptote	Identifies or describes the vertical and horizontal asymptotes of a given rational or exponential function with translations
9.2.1.8 **Make qualitative statements about the rate of change of a function based on its graph or table of values.**	Identifies a graphed positive linear slope as being correlated to constant increase in a context	Identifies that rate of change or changes in slope on a graph can represent changes in movement or speed in a given context Understands a horizontal line on a piecewise graph as meaning the output value is not changing over time	Identifies parts of a piecewise graph that have greatest or least average rate of change and represents the intervals in words or with inequality notation	Identifies changes in the rate of change over different intervals from points given in a table or graph Differentiates between linear and exponential representations of changes in movement or speed in a given context

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9.2.1.9 **Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations.**	Knows that a translation will move a given function from its current location to another location on the graph	Identifies how vertical translations change equations or graphs	Identifies the equation or graph of a new function based on a written description of applied dilations and/or translations and vice versa Graphs a new function based on a description of a translation of $f(x) = x^2$ **Released Example:** 500801	Graphs a new function based on translations of a given function in the form $f(x) = a(x - h)^2 + k$, where h and k are non-zero quantities Understands how translations affect equations to solve novel mathematical situations

Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context. (9.2.2)

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9.2.2.1 **Represent and solve problems in various contexts using linear and quadratic functions.**	Solves and identifies linear word problems in contexts such as with money, with whole-number answers Identifies a solution to a quadratic equation in the format $x^2 + c = d$ **Released Example:** 791064	Solves linear problems in contexts using whole numbers and decimals less than 10,000 and combining like information (terms) Identifies an equation to represent a linear situation given in context, without calculation Identifies the solution pair to a quadratic equation in the format $x^2 + bx + c = 0$ **Released Example:** 502015	Solves linear and quadratic problems with real-world and mathematical (e.g., area, perimeter) contexts using rational numbers Identifies an equation to represent a linear situation given in context and requiring calculations Identifies a solution to a quadratic equation in the format $ax^2 + bx + c = d$, where $a \neq 0$ or 1 **Released Example:** 502404	Creates linear and quadratic equations that represent the situations, combining multiple formulas when needed, to solve for an unknown variable **Released Examples:** 504016, 506210

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9.2.2.2 **Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.**	Solves a simple exponential growth equation for a given value	Solves for unknown variables given a simple exponential graph or function in context **Released Examples:** 501336, 501072	Creates, identifies, and/or solves the increasing or decreasing exponential equation in $y = ab^x$ format using the data given Identifies and/or solves initial or final value of simple compound interest equations, compounded annually or continuously, using given data **Released Examples:** 502363, 503184	Solves exponential equations representing changes in interest rate, initial value, or final value in context Identifies which exponential function represents a given situation of growth or depreciation **Released Example:** 500002

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9.2.2.3 **Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.**	Identifies the table of values that corresponds to a linear graph when the ordered pairs in the table are present on the graph Translates between a table of values for a linear function and the equation for the line Understands that the point (0, *b*) corresponds to the *y*-intercept of a graphed function	Given the graph of a linear function, computes the slope of the line and finds ordered pairs contained on the line which fall outside of the range of the graph shown Translates between graphs, tables, equations, and verbal descriptions of exponential functions in real-world and mathematical situations **Released Example:** 502163	Fluently translates between graphs, tables, equations, and verbal descriptions of linear, quadratic, and exponential functions, including when no information about the type of function has been provided	Sketches graphs of linear, quadratic, and exponential functions presented in novel forms, including functions for which translations or scale factors have been applied

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9.2.2.4 **Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively.**	Identifies the common ratio for a geometric sequence when the absolute value of the terms are increasing and when the common ratio is a whole number less than 5	Identifies the value of a designated term in a geometric sequence when given an explicit formula using only integers Identifies when an explicit formula produces a geometric sequence versus an algebraic sequence Knows a common ratio with an absolute value less than 1 will cause the absolute value of the values in a sequence to decrease instead of increase **Released Example:** 45376	Identifies the formula, explicit or recursive, of a given pattern shown in a list, table, graph, or image format Calculates the sum of up to 6 terms in a geometric series Identifies the common ratio of a given geometric series	Identifies the parts of a recursive and explicit function and solves for up to 4 additional terms in the sequence Identifies the recursive or explicit function that represents a given real-world or abstract mathematical situation

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9.2.2.5 **Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.**	Calculates consecutive terms in a finite geometric series given in context	Calculates a solution to problems up to the fifth term in a finite geometric series in context Calculates a solution to problems in context using the annual compound interest formula at a given percentage rate	Computes value of account after *n* years using the compound interest formula for various compounding periods Calculates the sum of a finite increasing or decreasing geometric series for up to 10 terms Represents a geometric sequence and/or finds terms of the sequence in contexts such as salary increase or depreciation when given initial value and percent increase/decrease	Solves real-world problems related to finding the total amount by computing the sum of a finite geometric series given as a verbal description Interprets beginning and ending time periods properly and differentiates when to use the exponents ($n - 1$) and n when computing terms of a geometric sequence in real-world contexts Computes the nth term of a finite increasing or decreasing geometric sequence in real-world context when given recursively or as a verbal description

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9.2.2.6 **Sketch the graphs of common non-linear functions such as *f(x) = √x, f(x) = |x|, f(x) = 1/x, f(x) = x^3* and translations of these functions, such as *f(x) = √(x—2) + 4*. Know how to use graphing technology to graph these functions.**	Identifies vertical translations of the parent square root, absolute value, and cubic functions	Identifies the general shape of a graphed square root or absolute value function	Identifies the equation of a function representing a given graph, or vice versa, when the function contains horizontal and/or vertical translation for rational, absolute value, square root, and cubic functions Identifies the graph or equation from a written description of one (horizontal) or two (horizontal and vertical) translations of a parent function Identifies the general shape of a graphed cubic and rational function	Understands the effect of adding or subtracting an abstract number "*n*" inside or outside the argument of a function and uses this information to translate the graph of a function to a specified quadrant

Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions. (9.2.3)

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9.2.3.1 **Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.**	Evaluates simple rational expressions or expressions containing radicals or absolute values for integer values of the variable	Evaluates multi-term expressions containing one or more rational expressions, absolute value expression, or radical expression and for rational number values of the variable	Consistently evaluates polynomial expressions with exponents of 2 or 3 and polynomial expressions that are the arguments of radical or absolute value expressions for integer values of the variable Understands that odd powers of negative numbers result in a negative number	Fluently evaluates multi-term polynomial expressions containing more than two rational expressions, absolute value, or radical expressions with multiple variables and/or multiple occurrences of the variable

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9.2.3.2 **Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.**	Multiplies single variable monomials and combines like terms when positive coefficients are larger than negative coefficients	Adds and subtracts polynomial expressions containing two or more terms Multiplies binomials of the form $(x \pm a)(x \pm b)$ **Released Example:** 503049	Divides polynomials using long division when the divisor is a linear polynomial such as $(ax + b)$	Fluently adds, subtracts, multiplies, and divides polynomials including when presented in novel forms
9.2.3.3 **Factor common monomial factors from polynomials, factor quadratic polynomials and factor the difference of two squares.**	Factors a monomial from a binomial **Released Example:** 502400	Factors quadratic polynomials of the form $x^2 + bx + c$ and difference of two squares	Factors trinomials of the form $ax^2 + bx + c$ and containing monomial common factors	Fluently factors polynomials presented in novel forms

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9.2.3.4 **Add, subtract, multiply, divide, and simplify algebraic fractions.**	Understands that dividing equivalent monomials results in a quotient of 1	Adds algebraic fractions with like denominators	Adds and subtracts two algebraic fractions with unlike single factor denominators to simplify Multiplies two algebraic fractions with linear term **Released Example:** 44803	Multiplies and divides algebraic fractions, including when numerators and denominators contain linear or quadratic expressions that have not been factored to simplify Adds and subtracts algebraic fractions with unlike multiple factor and/or multiple term denominators to simplify

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9.2.3.5 **Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers.**	Knows that imaginary numbers exist	Understands that some quadratic equations have two solutions that may contain the notation "i" and "±"	Substitutes a complex number of the form $\pm bi$ into a quadratic equation of the form $x^2 + c = 0$ to check if the numbers are solutions	Understands that complex number solutions to quadratic equations come in pairs Substitutes numbers of the form $a \pm bi$ into quadratic equations to check whether they are solutions Uses solution in $a \pm bi$ form of quadratic equation to solve for an unknown coefficient in quadratic equation

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9.2.3.6 **Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving *n*th roots.**	Applies the exponent product rule to simple equations and expressions with integer exponents	Understands that $\frac{1}{n} = n^{-1}$ Understands the denominator of a rational exponent as corresponding to a root in a radical expression	Uses the laws of exponents to simplify algebraic expressions involving positive and negative integer exponents with multiple variables and positive rational exponents	Consistently simplifies complex algebraic expressions involving multiple variables and/or containing both negative and positive rational exponents Generates equivalent expressions containing *n*th roots and multiplies radical expressions containing unlike roots

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9.2.3.7 **Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.**	Identifies the distributive property of the form *a(b + c)* to generate equivalent expressions of the form *ab + ac*	Identifies the distributive, commutative, and associative properties of equality to justify solution steps in generating equivalent expressions	Understands that when rational expressions are simplified, the original and simplified forms may not be equivalent for all values of the variable Uses the distributive, commutative, and associative properties of equality to generate equivalent expressions	Consistently differentiates between properties of equality and uses them to justify steps to simplify expressions

Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context. (9.2.4)

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9.2.4.1 **Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.**	Solves quadratic equations which yield integer solutions using factoring	Solves quadratic equations which yield integer solutions using substitution or factoring	Consistently solves quadratic equations in context using factoring and for which the solutions are rational numbers **Released Example:** 501096	Consistently solves quadratic equations presented in any form including using the quadratic formula Solves multi-step problems for which solving a quadratic equation is a step in the process Interprets solutions to real-world quadratic problems and recognizes when solutions may not be applicable

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9.2.4.2 **Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.**	Substitutes values into an exponential function to find the requested result **Released Example:** 506156	Computes input or output value of an exponential function when given the other	Uses an exponential function, in equation or graph form, to solve real-world problems **Released Example:** 503106	Models real-world exponential growth or decay situations, and uses them to solve problems

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9.2.4.3 **Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.**	Recognizes that some quadratic equations cannot be solved (with real numbers)	Understands that some quadratic equations have non-real solutions	Solves quadratic equations of the form $ax^2 + c = 0$ and interprets the square root of a negative number to mean that the solutions will be 2 non-real complex numbers of the form $\pm bi$	Solves quadratic equations in all forms and with solutions that are real and non-real ($a \pm bi$ form where *b* is not zero) Uses understanding of the structure of the quadratic formula and of quadratic equations to determine, without solving, the number and type (i.e., real or non-real) of solutions

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9.2.4.4 **Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.**	Understands that dashed lines graphed on a coordinate grid represent strict inequalities (< or >)	Verifies a solution point for a linear inequality within the shaded region	Identifies the system of linear inequalities symbolically that represents a described context Understands that when a system of linear inequalities is graphed on a coordinate grid, ordered pairs on a solid line are contained in the solution set while ordered pairs on a dashed line are not contained in the solution set Identifies the solution set for a system of linear inequalities graphically when given in symbolic form and when one of the two inequalities uses < or > and the other uses ≤ or ≥ Identifies a system of linear inequalities symbolically when given a graph of the linear inequalities when one of the lines is dashed and the other is solid **Released Example:** 506031	Graphs a system of linear inequalities from a written description in context or as symbolic inequalities

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9.2.4.5 **Solve linear programming problems in two variables using graphical methods.**	Identifies solutions to linear programming problems as points within the feasible region	Identifies maximum or minimum values as solutions to linear programming problems	Solves linear programming problems in two variables given graph of constraints, feasible region, and objective function	Solves mathematical and real-world linear programming problems in two variables given a verbal description or a list of constraints and objective function, including a mix of inequality types ($<$, $>$, $\leq$, and $\geq$) and when the slopes of the boundaries are different from 0 or undefined
9.2.4.6 **Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically.**	Identifies the shape of an absolute value graph	Identifies the graph of absolute value inequalities with vertical translation $\|x\| \pm c$	Identifies graph of an absolute value inequality in the form $\|x \pm b\| \pm c$	Identifies absolute value inequality symbolically given a verbal description or a graph of absolute value inequality Creates an absolute value inequality for a real-world or mathematical context

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9.2.4.7 **Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods.**	Solves simple square root equations (e.g., $\sqrt{(ax + b)} = c$) that do not have extraneous solutions	Solves an equation in the form $\sqrt{ax \pm b} = \sqrt{cx \pm d}$	Identifies solutions to equations containing radical expressions Identifies extraneous solutions when one or both solutions are negative **Released Example:** 45413	Consistently solves equations containing one or more radical expressions involving cube or square roots and recognizes extraneous solutions **Released Example:** 790524
9.2.4.8 **Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.**	Finds and interprets a solution to a problem in context that uses positive integers **Released Example:** 506034	Interprets solutions to linear and quadratic situations in context	Interprets the meaning of a point located on a graph or in an equation in original context, including exponential and quadratic situations **Released Example:** 506036	Explains why a given solution is reasonable in the provided context Interprets the meaning of a given point(*s*) of an equation or inequality in context of the meaning of the *x*- and *y*-axis

Geometry & Measurement

Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations. (9.3.1)

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9.3.1.1 **Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.**	Identifies the surface area of a sphere when given a radius less than 4 Identifies a reasonable estimate for the volume of a sphere from a list of disparate values Identifies the volume of a cone when given a formula and all dimensions	Finds the volume of a square pyramid when given a formula and all dimensions	Determines the surface area of a sphere when given the radius or diameter and vice versa Determines the volume of a small square pyramid when given all dimensions **Released Example:** 503475	Consistently works backwards to find the measure of an unknown dimension, including when the task involves a context Appropriately chooses between height and slant height when determining surface area of pyramids and cones

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9.3.1.2 **Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.**	Calculates the areas of triangles and rectangles given the dimensions Calculates the volume of rectangular prisms given the dimensions	Calculates the area of a composite figure that is shown decomposed into a rectangle and triangles	Determines area or perimeter of a composite figure that decomposes into rectangles, triangles, and/or parts of circles	Decomposes multiple two-dimensional figures represented in diagrams or described in words to find and compare areas and perimeters Decomposes three-dimensional figures represented in diagrams to determine surface area or volume, including when the figure involves at least one curved surface Identifies cross-sectional shapes of three-dimensional figures

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9.3.1.3 **Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.**	Solves one-step problems involving a basic conversion Knows that "square units" and "cubic units" are not used in linear measurements	Solves two-step unit conversion problems when both conversions are within the same measurement system and unit type (e.g., inches to feet to yards)	Solves two- and three-step unit conversion problems when the conversions are in different measurement systems or across two different unit types (e.g., quarts/minute to gallons/hour, minutes to days) Explains how to solve simple problems in words or with linear dimensional analysis that includes the appropriate unit labels **Released Example:** 45178	Solves conversion problems involving square and cubic units

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9.3.1.4 **Understand and apply the fact that the effect of a scale factor *k* on length, area and volume is to multiply each by *k*, k^2 and k^3, respectively.**	Understands that a given scale factor *k* will make lengths *k* times larger	Applies a given scale factor to a given dimension	Applies scale factor changes to areas of quadrilaterals and volumes of rectangular prisms	Applies scale factor changes to polygons and prisms Understands how to apply scale factor changes to different dimensions on figures involving curved sides (e.g., circles, cylinders, spheres)
9.3.1.5 **Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.**	Assessed within 9.3.1.1 through 9.3.1.4	Assessed within 9.3.1.1 through 9.3.1.4	Assessed within 9.3.1.1 through 9.3.1.4	Assessed within 9.3.1.1 through 9.3.1.4

Construct logical arguments, based on axioms, definitions and theorems to prove theorems and other results in geometry. (9.3.2)

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9.3.2.1 **Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.**	Assessed within 9.3.2.2 and 9.3.2.4	Assessed within 9.3.2.2 and 9.3.2.4	Assessed within 9.3.2.2 and 9.3.2.4	Assessed within 9.3.2.2 and 9.3.2.4
9.3.2.2 **Accurately interpret and use words and phrases such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive.**	Writes the sentence that reflects "$P \rightarrow Q$" given statements for P and Q	Identifies a true statement based on the transitive property	Identifies the inverse of a given statement **Released Example:** 790537	Identifies the converse and contrapositive of a given statement Identifies an equivalent statement to a given statement

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9.3.2.3 **Assess the validity of a logical argument and give counterexamples to disprove a statement.**	Assessed within 9.3.2.4	Assessed within 9.3.2.4	Assessed within 9.3.2.4	Assessed within 9.3.2.4
9.3.2.4 **Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations.**	Knows some of the common terms used (e.g., proof, given, definition, theorem)	Identifies congruent segments based on the definition of midpoint Knows that triangle congruence postulates can be used to show triangle congruence	Follows along a given 2-column or flow-chart proof to identify which triangle congruence postulate can be used in the proof	Understands how and when to use CPCTC and transitive properties Completes parts of triangle congruence proofs when the structure has been provided

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9.3.2.5 **Use technology tools to examine theorems, make and test conjectures, perform constructions and develop mathematical reasoning skills in multi-step problems. The tools may include compass and straight edge, dynamic geometry software, design software or Internet applets.**	Knows that a compass can be used to construct equal length segments **Released Example:** 502314	Knows that an arc must be used when constructing either an angle or perpendicular bisector	Identifies steps in the process of constructing either an angle or perpendicular bisector Determines congruence based on a construction process	Consistently carries out all necessary steps to construct a congruent figure or bisector

Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry. (9.3.3)

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9.3.3.1 **Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results.**	Finds measure of one alternate interior or same-side interior angle of another angle in a diagram Finds the measure of a complementary angle in a diagram involving perpendicular lines intersected by a third line	Applies up to two properties involving parallel and/or perpendicular lines to find an unknown angle measure in a diagram when known angle measures are whole numbers	Applies up to three properties involving parallel and/or perpendicular lines to find an unknown angle measure in a diagram, including when angles are represented by algebraic expressions	Applies at least four properties involving parallel and/or perpendicular lines to solve problems finding at least one unknown angle measure in a diagram

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9.3.3.2 **Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results.**	Uses properties of supplementary and/or corresponding angles to find an unknown angle measure in a diagram	Uses properties of vertical and/or complementary angles to find an unknown angle measure in a diagram Finds measure of one corresponding angle of another angle in a diagram	Applies multiple angle properties to solve multi-step problems presented with diagrams involving finding one or more unknown angle measures, including when angle measures are represented by algebraic expressions	Applies multiple angle properties to solve multi-step problems presented with verbal descriptions, finding more than one unknown angle measurement
9.3.3.3 **Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.**	Knows the definitions of equilateral, isosceles, and scalene triangles	Understands that the base angles opposite congruent sides are congruent Uses properties of equilateral triangles to find unknown angles or sides	Uses properties of isosceles triangles to find unknown angles or sides, including when the isosceles triangle is rotated	Consistently applies properties of triangles to solve multi-step problems involving either a complex diagram with more than one type of triangle or no diagram at all Consistently applies the relationship between angle measure and length of the side opposite the angle in scalene triangles

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9.3.3.4 **Apply the Pythagorean Theorem and its converse to solve problems and logically justify results.**	Recites "*a* squared plus *b* squared equals *c* squared"	Applies the Pythagorean Theorem to find an unknown side length in a right triangle **Released Examples:** 506099, 500682	Consistently interprets context to apply the Pythagorean Theorem and its converse to solve problems	Consistently interprets complex contexts and/or diagrams to apply the Pythagorean Theorem and its converse to solve multi-step problems
9.3.3.5 **Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results.**	Knows 45-45-90 and 30-60-90 triangles are right triangles	Knows 45-45-90 triangles are isosceles triangles and that two sides are the same length	Knows and applies properties of 45-45-90 and 30-60-90 triangles to solve problems involving one triangle	Knows and applies properties of 45-45-90 and 30-60-90 triangles to solve problems involving complex figures

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9.3.3.6 **Know and apply properties of congruent and similar figures to solve problems and logically justify results.**	Recognizes relationships of corresponding angles and sides in triangles when the triangles are specifically described as congruent or similar in words or with symbols, including when one triangle is a rotation of the other **Released Example:** 503097	Recognizes information needed for proving congruence or similarity (e.g., SAS and AA) Uses proportions to represent and find unknown side lengths in similar figures	Consistently uses equivalent ratios, not differences, to find unknown side lengths in similar figures described in words or displayed figures Consistently recognizes that angle measure remains the same in similar triangles **Released Example:** 503425	Uses multi-step reasoning involving properties to draw conclusions about the relationships represented in a geometric diagram when the diagram involves multiple and overlapping figures

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9.3.3.7 **Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.**	Uses properties of parallelograms and trapezoids to find unknown angle measures	Uses properties of parallelograms to find unknown lengths of diagonals	Fluently uses definitions and properties of polygons to solve problems involving angle measures or side lengths	Consistently solves problems involving the interior angles of regular polygons with or without a diagram Knows and applies the hierarchy of quadrilaterals Justifies statements about polygons

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9.3.3.8 **Know and apply properties of a circle to solve problems and logically justify results.**	Computes areas or circumferences of circles given radius or diameter in mathematical problems Finds diameters of circles when given circumferences	Knows that a tangent line to a circle forms a right angle with the radius at the point of tangency	Calculates measure of a central angle given the inscribed angle and vice versa Recognizes that a triangle formed with a central or an inscribed angle as its apex is isosceles Uses proportional reasoning and knowledge of area and circumference to calculate areas of sectors and lengths of arcs	Applies multiple properties of a circle to solve problems involving multiple and overlapping figures (e.g., opposite angles of an inscribed quadrilateral are supplementary, tangents drawn from a point collinear with the center to two different radii are congruent, and an inscribed angle that intersects a diameter at its endpoints forms a right triangle) **Released Example:** 502407

Solve real-world and mathematical geometric problems using algebraic methods. (9.3.4)

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9.3.4.1 **Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.**	Uses the Pythagorean Theorem to find a whole-number, unknown side length in a right triangle	Identifies the trigonometric values of a given angle from a labeled right triangle Uses the definitions of trigonometric ratios to find the side lengths of similar triangles	Knows similar triangles have the same trigonometric ratios for corresponding angles Knows and uses the trigonometric relationships between the two acute angles in a right triangle	Finds unknown side lengths of similar right triangles, with or without a labeled diagram given, and then solves for trigonometric ratios Solves for other trigonometric ratios in a right triangle when one ratio is given in decimal format

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9.3.4.2 **Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.**	Knows sine, cosine, and tangent involve ratios from triangles	Sets up an equation to find an unknown side length of a triangle using a trigonometric ratio and can solve if needing only multiplication **Released Example:** 503444	Uses trigonometric ratios to solve for unknown side lengths or areas for real-world problems that can be decomposed into right triangles Knows that inverse trigonometric functions can be used to find angle measurements **Released Example:** 503445	Uses multiple trigonometric ratios to find lengths of unknown parts of segments involving right triangles for real-world problems Uses trigonometric ratios to solve for unknown side lengths of right triangles for real-world problems in three dimensions

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9.3.4.3 **Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.**	Assessed within 9.3.4.1 and 9.3.4.2	Assessed within 9.3.4.1 and 9.3.4.2	Assessed within 9.3.4.1 and 9.3.4.2	Assessed within 9.3.4.1 and 9.3.4.2
9.3.4.4 **Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.**	Finds the length of a horizontal or vertical line segment given coordinates of the endpoints or when shown on a grid	Uses verbal description of points on a line segment to identify midpoint, endpoints, or other points on the line Finds coordinates of the midpoint of a line segment given the coordinates of the endpoints	Calculates the length or slope of a line segment given coordinates of the endpoints or when shown on a grid Finds coordinates of one endpoint of a line segment given the other endpoint and the midpoint **Released Example:** 502085	Calculates length and slopes of segments using midpoint formula, distance formula, and properties of polygons (e.g., parallelogram, rhombus, square)

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9.3.4.5 **Know the equation for the graph of a circle with radius *r* and center (*h, k*), $(x-h)^2 + (y-k)^2 = r^2$, and justify this equation using the Pythagorean Theorem and properties of translations.**	Identifies the coordinates of the center of a circle when it falls on a lattice point in a coordinate grid Identifies the radius of a circle as being the distance from the center point to the edge of the circle	Uses the Pythagorean Theorem to find the hypotenuse length of right triangles given two leg lengths	Identifies the equation of a circle given the center and the radius Finds the center and the radius given the equation of a circle	Determines the equation of circle from its graph on a coordinate grid Finds the radius and equation of a circle given the center point and another point on the circle

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9.3.4.6 **Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate grid.**	Identifies the translation that moves point *A* to point *B* on a grid	Translates polygons on a grid given a rule of the form $(x, y) \rightarrow (x + a, y + b)$ Identifies the type of transformations used given a shape and its image	Finds coordinates of points when scaling (dilation centered at the origin) Demonstrates understanding of how reflections over the *x*- and *y*-axis change points on a graph and finds the distance between a point and its reflection	Rotates shapes about the origin by multiples of 90 degrees to find location of images Finds coordinates of image or preimage, when given the other, of polygons or lines which have undergone multiple transformations, including scale changes, rotations, reflections across lines other than the axes, and translations Identifies the rule used for a compound transformation (i.e., involving more than one basic transformation)

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9.3.4.7 **Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.**	Computes areas and volumes of basic shapes when provided with all dimensions	Uses the Pythagorean Theorem to find the hypotenuse length of right triangles given two leg lengths	Uses properties of triangles, parallelograms, and similar triangles to solve for unknown angle measures and side lengths Uses the Pythagorean Theorem to solve for unknown side lengths **Released Example:** 502089	Uses properties of parallel lines, squares, rectangles, triangles, and cylinders to solve for unknown side lengths, perimeter, angle measurements, area, or volume Creates equivalent expressions for area or volume of similar triangles, rectangles, and cylinders

Data Analysis & Probability

Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships. (9.4.1)

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9.4.1.1 **Describe a data set using data displays, including box-and-whisker plots; describe and compare data sets using summary statistics, including measures of center, location and spread. Measures of center and location include mean, median, quartile and percentile. Measures of spread include standard deviation, range and inter-quartile range. Know how to use calculators, spreadsheets or other technology to display data and calculate summary statistics.**	Identifies the minimum and maximum values from a labeled box-and-whisker plot **Released Example:** 504044	Understands the meanings of mean, median, and quartile values Identifies the mean, median, maximum, minimum, quartiles, and range from data with up to 10 whole-number values to find solutions Finds requested values for median and quartile values from box-and-whisker plots Uses the structure of a box-and-whisker plot to answer questions **Released Example:** 503502	Calculates, describes, and compares summary statistics Creates a basic data set with specified measures of center and spread Demonstrates understanding of the meaning of interquartile range compared to other measures of center, location, and spread in a box-and-whisker plot	Calculates summary statistics, including standard deviation, to analyze and interpret data given in box-and-whisker plots, stem-and-leaf plots, tables, bar graphs, and lists Solves problems using the relationship between quartiles and percentages within data sets Understands the effects of outliers on means and medians, and from that decides which measure of center is best to use in situations Creates a box-and-whisker plot of given data **Released Example:** 503543

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9.4.1.2 **Analyze the effects on summary statistics of changes in data sets.**	Knows that measures of center can change with the removal or addition of data	Calculates a new mean when additional data is added to a data set	Calculates additional data values that could be added to a given data set to obtain a specific mean Knows that multiplying the same constant to all values of a data set does not change percentile standings Calculates or explains how the mean, median, or range change when additional data is added to a data set	Calculates or explains how multiple summary statistics change when data is removed and/or additional data is added to a data set Knows that adding the same constant to all values of a data set does not change measures of spread Understands the relationship between standard deviation and range **Released Example:** 790792

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9.4.1.3 **Use scatterplots to analyze patterns and describe relationships between two variables. Using technology, determine regression lines (line of best fit) and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.**	Makes predictions in context within a range of reasonable values	Describes the relationship between variables in context displayed in a scatterplot Uses a line of best fit to make predictions in context	Finds the slope of a line of best fit displayed on a scatterplot Describes a possible correlation coefficient, including direction and strength, from a graph and describes the relationship between the two variables in context based on the correlation coefficient Identifies the type of function modeled by the data in the scatterplot (e.g., linear, quadratic, exponential)	Uses technology to identify the regression line from data given in a table or scatterplot Identifies how removal of a data point could affect the correlation coefficient, the reliability of the data, and/or the slope of the line of regression Describes how the value of the correlation coefficient affects the reliability of a prediction based on a regression line

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9.4.1.4 **Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.**	Calculates mean of a small data set when given in a list or table with whole numbers or decimals to the hundredths place	Identifies the mean when given a normal distribution (bell-shaped curve)	Identifies 1 standard deviation below and above the mean	Knows and applies the "68-95-99.7 Rule" to estimate percentages, number of data points, and data intervals when given the sample size, mean, and standard deviation of a normally distributed data set Calculates the standard deviation of a data set when given the mean, normal distribution intervals, and/or percentages for normally distributed data Estimates the area under a normal curve

Explain the uses of data and statistical thinking to draw inferences, make predictions, and justify conclusions. (9.4.2)

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9.4.2.1 **Evaluate reports based on data published in the media by identifying the source of the data, the design of the study and the way the data are analyzed and displayed. Show how graphs and data can be distorted to support different points of view. Know how to use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays**	Not assessed on MCA-III	Not assessed on MCA-III	Not assessed on MCA-III	Not assessed on MCA-III

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9.4.2.2 **Identify and explain misleading uses of data; recognize when arguments based on data confuse correlation and causation.**	Knows that comparing heights in a bar graph should only be done if the axis starts at 0 Knows data gathered from a subgroup cannot necessarily generalize to a larger population **Released Example:** 502224	Knows that a break in an axis on a graph can mislead the reader	Identifies how changes in vertical and horizontal scales can affect presentation of the data Understands the difference between causation and correlation	Understands and explains when data sets show a relationship or no relationship between correlation and/or causation
9.4.2.3 **Design simple experiments and explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.**	Understands that increasing sample size can make survey results more reliable	Identifies that a larger sample size reduces margin of error	Identifies bias in survey questions Understands that representative samples are needed to reduce bias and that random selection from a population reduces bias	Describes how a sampling method can be changed to avoid measurement errors

Calculate probabilities and apply probability concepts to solve real-world and mathematical problems. (9.4.3)

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9.4.3.1 **Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.**	Uses the addition principle to find the size of the sample space Multiplies number of outcomes to find the size of the sample space in simple problems **Released Example:** 503337	Uses the multiplication principle to find the size of the sample space of compound events	Uses the multiplication principle to find the probability of two events	Consistently applies counting procedures in real-world situations when determining the size of the sample space and probabilities Distinguishes situations in which order matters with those in which order does not matter and understands when to use replacement versus non-replacement when computing probabilities or number of possible outcomes **Released Examples:** 502371, 502138

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9.4.3.2 **Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.**	Knows that experimental probability is the number of successes divided by the total number of trials	Computes theoretical probability for simple, familiar, real-world situations Computes experimental probabilities using frequency tables with 6 or fewer categories and uses these to make predictions	Distinguishes between theoretical probability and experimental probability Calculates experimental probabilities using simulations or given data frequencies **Released Example:** 500158	Calculates experimental probabilities of compound events using simulations or given data frequencies **Released Example:** 500219
9.4.3.3 **Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.**	Calculates the experimental and theoretical probabilities of an event	Knows that experimental probabilities improve with more trials	Explains that the Law of Large Numbers means experimental probabilities will approach theoretical probabilities as the number of trials increases	Uses results of simulations or experiments and the Law of Large Numbers to estimate actual probabilities and percentages

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9.4.3.4 **Use random numbers generated by a calculator or a spreadsheet, or taken from a table, to perform probability simulations and to introduce fairness into decision making.**	Uses random numbers from simple simulations to provide experimental probabilities Knows to disregard random numbers outside range of values needed to conduct a simulation	Follows instructions to use random digits to conduct a simulation **Released Example:** 45327	Understands that randomly generated numbers are equally likely to occur Uses a random digit table to conduct simulations, with and without using repeated values Uses random numbers to make decisions about equally likely events	Correctly assigns values for random numbers to indicate specific outcomes in simulations

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9.4.3.5 **Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems.**	Finds the probability of a simple independent event	Finds the probability of a single event that meets two conditions	Recognizes that to compute the probability of an intersection of two events requires multiplying the probabilities Understands that a reduction in the size of the sample space has an impact on the computation of probability for compound events Creates a probability model that models stated probabilities for compound events	Recognizes the difference between dependent and independent events and computes conditional probabilities for both independent and dependent events Understands that to find the probability for a union of several events requires addition of probabilities or the use of binomial coefficients and uses the probability of the complement to compute unions efficiently Uses Venn diagrams to find probabilities of intersections, unions, and complements of events **Released Example:** 503236

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9.4.3.6 **Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.**	Understands the overlap in a Venn diagram with two circles as meaning membership in both sets	Completes a double Venn diagram when given sufficient information Understands objects outside of both circles on a Venn diagram as meaning not a member of either set	Uses multiple relational words AND, OR, and NOT to solve problems involving Venn diagrams	Recognizes and uses symbols for union and intersection to determine regions of membership in double or triple Venn diagrams **Released Example:** 790507
9.4.3.7 **Understand and use simple probability formulas involving intersections, unions and complements of events.**	Knows how to compute theoretical probabilities of simple events as a ratio	Finds probabilities of complements of simple events **Released Example:** 501380	Finds the probability of two simple independent events by multiplying individual probabilities (uses the multiplication principle)	Finds probabilities of compound events involving intersections, unions, and/or complements, including for non-disjoint and dependent events

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9.4.3.8 **Apply probability concepts to real-world situations to make informed decisions.**	Calculates the experimental probability of an event	Computes expected values from given probabilities or percentages **Released Example:** 503175	Uses data in a table or Venn diagram to compute experimental probabilities, including probabilities involving unions and intersections, to make decisions in real-world situations	Finds conditional probabilities and combinations for compound problems
9.4.3.9 **Use the relationship between conditional probabilities and relative frequencies in contingency tables.**	Understands that the sample size is reduced when computing a conditional probability	Computes conditional probabilities using contingency tables when totals are provided	Identifies and computes conditional probabilities as a fraction or decimal, using contingency tables when totals are not provided	Finds missing values in contingency tables given conditional probabilities